Game Theory Elective

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0 Preamble: About this class

Remark 0.1 (Background and motivation of this course)
With these classes, I want to achieve at least four goals:

1. Most immediately, it should provide a **gentle introduction to game theory**. Students will learn how to interpret real decision problems as a *game*. Then they will learn how to identify solutions to these decision problems (e.g., to compute the game's *equilibria*), and to simulate learning in games.
2. More generally, the course should serve as an **example of mathematical modeling**. Students should have in mind that the framework of game theory (the formalism, the solution concepts) are human constructs. They should understand why smart people at some point found it useful to come up with this formalism; they should be able to critically question the underlying assumptions and methods. Ideally, when taking this course, students should put themselves into the shoes of a smart social scientist, a hundred years ago (when game theory was not yet an established formalism). Ideally, they try to come up with some of the formalism and some of the solution concepts themselves.
3. I hope the course will also just serve as **brainfood**. Students should enjoy to hear more about some scientific discipline that they might find stimulating, but that they would not hear about otherwise.
4. It should give you some impression of the research we do in our research group; this might become relevant once you decide on your **master thesis**.

Remark 0.2 (Prerequisites)
A fully rigorous game theory course requires quite a bit of math (e.g., matrices, derivatives, fixed point theorems). However, within this course we will focus on intuitions. We introduce some key concepts without much mathematical formalism. For students who are interested in seeing more details and math, I can suggest some relevant reading material (e.g. Fudenberg and Tirole, 1998, Sigmund, 2010).

Remark 0.3 (Some administrative remarks)

- Overall structure. The course consists of eight lectures. These lectures will tell you about what game theory is about, how one can translate real-world interactions into games, and how to 'solve' these games (where 'solving' could be interpreted as: coming up with a prediction of how certain people might play this game). As you will see, the specific solution might depend on what type of game is played, and what assumptions we make on individuals' rationality (more on that later).
- How to "pass" this part of the course. There will be in-class exercises (as a group), online assignments (individually), and a final conversation/interview (for which I will post a list of possible questions).

1 An overview on game theory

Remark 1.1 (What is game theory?)
Game theory is a scientific field that explores strategic decision-making in interactions with multiple decision-makers who jointly determine each others' outcomes. In particular, not all decision-making is covered by game theory; as an example, the following questions are not covered:

How to play roulette? Should I buy a given flat at a given price? Should I do a PhD?

Instead, a strategic component is important. So, the following variations of the previous questions are in fact covered by game theory:

How to play poker? How to bargain the price of a flat you want to buy? How to apply to a PhD program?

In the following, we will ask two questions: (1) How can we formalize such strategic interactions in a mathematical model? (2) Given we have a model, what does it mean to *solve* a game?

In-Class Exercise 1.2 (Some examples to form intuitions)
Are the following questions covered by game theory (and if so, why?): (1) *Should I actively participate in this course?* (2) *Should I be modest or bold?* (3) *What should I wear today?*

Example 1.3 (Guess 2/3 of the average)
Let's play the following game. Everyone secretly writes down a number between 0 and 100. Then we compute the average of all numbers. Whoever is closest to 2/3 of this average, wins 2 EUR.

Remark 1.4 (Elements of a game)
When formalizing strategic interactions as a game, it is important to capture the following five elements:

- Players. Who are the relevant individuals that make (strategic) decisions?
- Actions. Between which actions can the players decide?
- Order of moves. Who gets to make a decision at which point?
- Information. What is it that players know when making their decisions?
- Payoffs. Depending on the players' decisions, what is the eventual outcome for each player? [e.g., measured in money, or happiness, or *utility*]

Example 1.5 (Revisiting auctions)
Suppose there is a piece of art by a famous artist that you might want to buy. You know the value that this piece of art has to you.. You don't know the value that other potential buyers assign to it. But you know how many other buyers there are, and you know that for each of them, their evaluation of the piece of art is randomly drawn from $\{0 \text{ EUR}, 1 \text{ EUR}, 2 \text{ EUR}, \ldots, 10 \text{ EUR}\}$. The rules of the auction are: Everyone secretly writes down the price they would be willing to pay (their *bid*). The potential buyer with the highest bid needs to pay the respective price, but gets the piece of art in return. (If there are multiple

buyers with the same bid, it's randomly allocated). So for example, if you happen to assign a value of 8 EUR to the piece of art, and if you have the highest bid (which happens to be 5 EUR), you will have a profit of 3 EUR. For this example of an auction, how would we define each of these five elements?

- *Players.* All the potential bidders, $\mathcal{N} = \{1, 2, \ldots, n\}$.
- *Actions.* Each player needs to name a price they are willing to pay, $p_i \in \{0, 1, \ldots, 10\}$ for $i \in \mathcal{N}$.
- *Order of moves.* All players need to make their decision simultaneously.
- *Information.* Players know how their own private value of the good, $v_i \in \{0, 1, \ldots, 10\}$. They don't know the precise value of the other players, but they know it's uniformly distributed in $\{0, 1, \ldots, 10\}$.
- *Payoffs.* The player who gets the item has a payoff of $v_i - p_i$; all others have a payoff of zero.

In-Class Exercise 1.6 (Split or Steal)
Split or Steal used to be the final stage of the UK game show *Golden Balls*. It involves two participants, each having two balls, marked 'Split' and 'Steal'. Participants independently choose one of the balls. If they both split, they share the prize money. If one splits and the other steals, the latter gets all the prize money. If they both steal, no one gets anything. Prior to making a decision, participants are allowed to talk to each other. Some examples:
`https://www.youtube.com/watch?v=bJzsYLSPY7o`
`https://www.youtube.com/watch?v=S0qjK3TWZE8&t=159s`
How would you define each of the five elements of this game?

Remark 1.7 (An overview on different kinds of game theories)
There are several different approaches to game theory:

1. Classical/Standard game theory. This kind of game theory asks what rational players would do. Being rational here means that players completely understand all aspects of the game, and that they wish to maximize their payoffs.
2. Evolutionary game theory. Here, players do not need to be rational. Rather they are myopic, but they try to improve their payoffs over time.
3. Epistemic game theory. This kind of game theory tries to understand how different assumptions on what people know shapes the predictions of game theory (e.g., if players are no longer assumed to be rational, or if players no longer believe in their co-players' rationality).
4. Algorithmic game theory. Here, researchers are concerned with the computational complexity of solving games, and in identifying optimal algorithms for doing so.
5. Experimental game theory. Here the question is: How do actual people behave in strategic interactions?

We will mostly focus on classical game theory, but we'll also have a brief look at evolutionary and experimental game theory (that's also what we do in our research group).

Remark 1.8 (Different classes of games)
There are various ways how to classify games. One common distinction is whether all players decide

once and simultaneously, or whether they make (possibly multiple) decisions over time (think of chess, for example). The respective game classes are called *static games* and *dynamic games*, respectively. Another distinction is whether players know all relevant aspects of the game. Here, the distinction is between *games with complete information* and *games with incomplete information* (think of chess vs poker).

2 Static games with complete information

2.1 Normal-form games

Remark 2.1 (**Normal-form games**)
In the following, we start by discussing the simplest possible case: static games with complete information among two players (who can choose among finitely many actions). That is, players make only one decision, and they need to decide simultaneously. When making their decision, they know all relevant aspects of the game (e.g., the possible actions and the feasible payoffs). The only thing they do not know is the decision of the other players. These games are called *normal form games*. They can be represented by a table (*payoff matrix*).

Example 2.2 (**Split or Steal**)
Let's revisit the game 'Split or Steal'. There are two players, who can choose among two actions. Depending on the players' actions, they either share the prize P, one of the players gets all, or they both get nothing. Ignoring the pre-game communication state, the situation can be represented by the table

	Split	Steal
Split	$P/2, P/2$	$0, P$
Steal	$P, 0$	$0, 0$

Here, one of the players (the blue player) chooses a row of the matrix and the other (the red player) chooses a column. The entries of the table represent the payoff of the respective player. Players wish to maximize their realized payoff.

In-Class Exercise 2.3 (**Deriving payoff matrices: Snowdrift game, stag-hunt game**)

1. There is a task that needs to be done (e.g., washing the dirty dishes in the kitchen sink). There are two individuals who need to decide whether or not to volunteer for doing the task. Both individuals prefer that the task is done, but they both prefer the other one to do it. Assume if at least one person volunteers, both get a benefit b. However, anyone who volunteers pays some small cost $c < b$. If both players volunteer, they share the cost.

2. There are two members of a tribe who need to decide whether to hunt stag or hare. To successfully hunt a stag, it takes two committed hunters. In that case, each hunter gets a payoff of 10. All by yourself, you cannot get a stag and you would get a payoff of 0 if you tried. Hunting a hare can be

done individually, without any risks, leading to a safe (but smaller) payoff of 4, independent of the other tribe member.

Remark 2.4 (Solving games)
So far, we have encountered several instances of strategic interactions, and we have translated them into *games*. Now that we have this game representation, we ask ourselves: How can we predict what *rational* individuals would do in such games? Or to put differently, how would those individuals *solve* these games? In the following, we discuss two solution concepts: *elimination of dominated actions* and the *Nash equilibrium*.

References

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