Modeling Human Behavior

An Introduction to game theory – Part 1: What are games?

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0 Preamble: About this class

Remark 0.1 (**Introductory discussion**)

1. What have your previous classes in the PIC *Modeling Human Behavior* been about?
2. Who already knows some game theory? What is it about, informally?
3. What is your pre-specialization project about? Does it involve some game theory?

Remark 0.2 (**Background and motivation of this course**)
With these classes, I want to achieve at least four goals:

1. Most immediately, it should provide a **gentle introduction to game theory**. Students will learn how to interpret real decision problems as a *game*. Then they will learn how to identify solutions to these decision problems (e.g., to compute the game's *equilibria*), and to simulate learning in games.
2. More generally, the course should serve as an **example of mathematical modeling**. Students should have in mind that the framework of game theory (the formalism, the solution concepts) are human constructs. They should understand why smart people at some point found it useful to come up with this formalism; they should be able to critically question the underlying assumptions and methods. Ideally, when taking this course, students should put themselves into the shoes of a smart social scientist, a hundred years ago (when game theory was not yet an established formalism). Ideally, they try to come up with some of the formalism and some of the solution concepts themselves.
3. I hope the course will also just serve as **brainfood**. Students should enjoy to hear more about some scientific discipline that they might find stimulating, but that they would not hear about otherwise.
4. It should give you some impression of the research we do in our research group; this might become relevant once you decide on your **master thesis**.

Remark 0.3 (**Prerequisites**)
A fully rigorous game theory course requires quite a bit of math (e.g., matrices, derivatives, fixed point theorems). However, within this course we will focus on intuitions. We introduce some key concepts without much mathematical formalism. Students who are interested in seeing more details (and some math) will learn more in the elective on game theory (in fact, our classes will be a teaser to that course). For even more math, or even more details, I will also suggest some relevant reading material.

Remark 0.4 (**Some administrative remarks**)

- Overall structure. The course consists of five parts. Today we discuss what game theory is about, and how to model real life interactions as games. Next week, we describe how to *solve* such games, either with classical equilibrium methods or with an evolutionary perspective. In June we will have two further classes that focus on some current research in game theory (in particular, on human behavior in social dilemmas).
- How to "pass" this part of the course. Grading will be based on participation (solving group exercises), on online assignments, and based on a final conversation/interview (for which I will post a list of possible questions).

1 A primer in game theory

1.1 What are games and what is game theory?

Remark 1.1 (**What is game theory?**)
Game theory is a scientific field that explores strategic decision-making in interactions with multiple decision-makers who jointly determine each others' outcomes. In particular, not all decision-making is covered by game theory; as an example, the following questions are not covered:

How to play roulette? Should I buy a given flat at a given price? Should I do a PhD?

Instead, a strategic component is important. So, the following variations of the previous questions are in fact covered by game theory:

How to play poker? How to bargain the price of a flat you want to buy? How to apply to a PhD program?

In the following, we will ask two questions: (1) How can we formalize such strategic interactions in a mathematical model? (2) Given we have a model, what does it mean to *solve* a game?

In-Class Exercise 1.2 (**Some examples to form intuitions**)
Are the following questions covered by game theory (and if so, why?): (1) *Should I actively participate in this course?* (2) *Should I be modest or bold?* (3) *What should I wear today?*

Example 1.3 (**A further example to form intuitions: auctions**)
Let's play the following game. Suppose there is a piece of art by a famous artist that you might want to buy. You know the value that this piece of art has to you (it is written on a Post-It). You don't know the value that other potential buyers assign to it. But you know how many other buyers there are, and you know that for each of them, their evaluation of the piece of art is randomly drawn from {0 EUR, 1 EUR, 2 EUR, . . ., 10 EUR}. The rules of the auction are: Everyone secretly writes down the price they would be willing to pay (their *bid*). The potential buyer with the highest bid needs to pay the respective price, but gets the piece of art in return. (If there are multiple buyers with the same bid, it's randomly allocated). So for example, if you happen to assign a value of 8 EUR to the piece of art, and if you have the highest bid (which happens to be 5 EUR), you will have a profit of 3 EUR. *I pay this profit in cash.*

Question: *How much are you willing to bid?* After the game: *How did you choose your strategies?*.

Remark 1.4 (Elements of a game)
When formalizing strategic interactions as a game, it is important to capture the following five elements:

- Players. Who are the relevant individuals that make (strategic) decisions?
- Actions. Between which actions can the players decide?
- Order of moves. Who gets to make a decision at which point?
- Information. What is it that players know when making their decisions?
- Payoffs. Depending on the players' decisions, what is the eventual outcome for each player? [e.g., measured in money, or happiness, or *utility*]

Example 1.5 (Revisiting auctions)
For our initial example of an auction, how would we define each of these five elements?

- *Players.* All the members of the classroom, $\mathcal{N} = \{1, 2, \ldots, n\}$.
- *Actions.* Each player needs to name a price they are willing to pay, $p_i \in \{0, 1, \ldots, 10\}$ for $i \in \mathcal{N}$.
- *Order of moves.* All players need to make their decision simultaneously.
- *Information.* Players know how their own private value of the good, $v_i \in \{0, 1, \ldots, 10\}$. They don't know the precise value of the other players, but they know it's uniformly distributed in $\{0, 1, \ldots, 10\}$.
- *Payoffs.* The player who gets the item has a payoff of $v_i - p_i$; all others have a payoff of zero.

In-Class Exercise 1.6 (Split or Steal)
Split or Steal used to be the final stage of the UK game show *Golden Balls*. It involves two participants, each having two balls, marked 'Split' and 'Steal'. Participants independently choose one of the balls. If they both split, they share the prize money. If one splits and the other steals, the latter gets all the prize money. If they both steal, no one gets anything. Prior to making a decision, participants are allowed to talk to each other. Some examples:
`https://www.youtube.com/watch?v=bJzsYLSPY7o`
`https://www.youtube.com/watch?v=S0qjK3TWZE8&t=159s`
How would you define each of the five elements of this game?

Remark 1.7 (An overview on different kinds of game theories)
There are by now several different approaches to game theory:

1. Classical/Standard game theory. This kind of game theory asks what rational players would do. Being rational here means that players completely understand all aspects of the game, and that they wish to maximize their payoffs.
2. Evolutionary game theory. Here, players do not need to be rational. Rather they are myopic, but they try to improve their payoffs over time.

3. Epistemic game theory. This kind of game theory tries models how individuals form expectations/beliefs if players are no longer assumed to be rational, or if players no longer believe in their co-players' rationality.

4. Algorithmic game theory. Here, researchers are concerned with the computational complexity of solving games, and in identifying optimal algorithms for doing so.

5. Experimental game theory. Here the question is: How do actual people behave in strategic interactions?

We will mostly focus on classical game theory, but we'll also have a brief look at evolutionary and experimental game theory (that's also what we do in our research group).

1.2 Normal-form games

Remark 1.8 (Normal-form game)
There are various ways how to classify games. For example, one common distinction is whether players decide once and simultaneously, or whether they make (possibly multiple) decisions over time. The respective game classes are called *static games* and *dynamic games*. Another distinction is whether players know all aspects of the game. So the distinction is between *games with complete information* and *games with incomplete information*. For most of this course, we will focus on *static games with complete information*. To get intuitions, we will look at the simplest possible case. We consider a scenario with two players only. Those players have complete knowledge of the possible actions and each other's possible payoffs. They each make one decision, and they need to decide simultaneously. These are so-called *normal form games*. They can be represented by a table (*payoff matrix*).

Example 1.9 (Split or Steal)
Let's revisit the game 'Split or Steal'. There are two players, who can choose among two actions. Depending on the players' actions, they either share the prize P, one of the players gets all, or they both get nothing. Ignoring the pre-game communication state, the situation can be represented by the table

	Split	Steal
Split	$P/2, P/2$	$0, P$
Steal	$P, 0$	$0, 0$

Here, one of the players (the blue player) chooses a row of the matrix and the other (the red player) chooses a column. The entries of the table represent the payoff of the respective player. Players wish to maximize their realized payoff.

In-Class Exercise 1.10 (Deriving payoff matrices, part I: Volunteer's dilemma)
There is a task that needs to be done (e.g., providing first aid after an accident, or washing the dirty dishes in the kitchen sink). There are two individuals who need to decide whether or not to volunteer for doing the task. Both individuals prefer that the task is done, but they would prefer the other one to do it. Assume if at least one person volunteers, both get a benefit b. However, anyone who volunteers pays some small cost c.

Homework Exercise 1.11 (Deriving payoff matrices, part II: Stag-Hunt game)
There are two members of a tribe who need to decide whether to hunt stag or hare. To successfully hunt a stag, it takes two committed hunters. In that case, each hunter gets a payoff of 10. All by yourself, you cannot get a stag and you would get a payoff of 0 if you tried. Hunting a hare can be done individually, without any risks, leading to a safe (but smaller) payoff of 4, independent of the other tribe member.

1.3 Summary and outlook

Remark 1.12 (A summary)
Today you should have learned

1. what game theory is about – about strategic decision-making when individuals need to take others' possible decisions or reactions into account.
2. how to formalize a game in a (mathematical) model – by identifying the players, their actions, the order of moves, the information they have, and the payoffs.
3. how to derive payoff matrices in the most simple case of a game, simultaneous-move games with complete information among two players ('normal form games').

Remark 1.13 (An outlook on the next class: Solving games)
So far, we looked at how we can represent strategic interactions in more formal way. Next, we ask: What does it mean to solve such games (roughly: How should we predict what rational players would do?) To this end, we will discuss two solution concepts: (1) (Iterated) elimination of dominated actions, and (2) Nash equilibria. For example: Think about: What do you think rational players should do in the Split or Steal game in Example 1.9?

Group discussion 1.14 (Possible further exercises)

- Discuss from a game theory viewpoint: groupwork; people's incentives to use (generative) AI; strategic interactions in literature or in movies; the competition between Boeing and Airbus.
- Volunteer's timing dilemma: Suppose the volunteer's dilemma is played over two periods (an early period and a late period). Individuals can decide to volunteer in either period, or not at all. Now, some individuals might want to strategically procrastinate their own volunteering to the late period – if they see their co-player volunteers to do the task in the early period, they can save the volunteering costs altogether. Can you come up with a payoff matrix for this scenario?